SOURCE SEPARATION IN SHORT IMAGE SEQUENCES USING DELAYED CORRELATION

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ABSTRACT

This paper addresses the use of blind sources separation techniques for analysis of short image sequences (multi-dimensional signals). We suggest a modification of Molgedey and Schuster's [8] approach allowing for an arbitrary number of source signals. The viability of the new algorithm is illustrated on a toy-problem image sequence separation problem.

1. INTRODUCTION

Principal Component Analysis (PCA) is a very popular tool for analysis of image databases, see e.g., [6]. By PCA the image database is decomposed in terms of "eigenimages" that often lend themselves to direct interpretation. A most striking example is face recognition where so-called "eigenfaces" are used as orthogonal preprocessing projection directions for pattern recognition. The Principal Components, viz. the sequence of projections of the image data onto the eigenimages, are also uncorrelated; hence, perhaps the simplest example of so-called *Independent Components* [5]. The basic tool for PCA is Singular Value Decomposition (SVD). Define $X = \{X_{j,t}\}$ the data matrix with $\dim(X) = (P, N)$. $X_{j,t}$ represents a multi-channel signal with $j = 1, \cdots, P$ denoting the pixel (channel) index and $t = 1, \cdots, N$ the time index. In SVD, X is decomposed as

$$X = UDV^\top, \quad X_{j,t} = \sum_{\ell=1}^{L} U_{j,\ell} D_{\ell,\ell} V_{t,\ell}, \qquad (1)$$

where $\dim(U) = (P, L)$, $\dim(D) = (L, L)$, and $\dim(V) = (N, L)$ and $L = \min(N, P)$. U, V are the orthonormal basis vectors (i.e., eigenvectors of the symmetric matrices $XX^\top$ and $X^\top X$, respectively) and D is a (L, L) diagonal matrix of singular values. In terms of independent sources, SVD can identify a set of uncorrelated time sequences: the Principal Components, $Y_{\ell,t} = D_{\ell,\ell} V_{t,\ell}$, enumerated by the source index ℓ. This allows us to write the observed image sequence as a weighted sum of fixed (eigen-)images $U_{j,\ell}$ (enumerated by ℓ), $X_{j,t} = \sum_{\ell=1}^{L} U_{j,\ell} Y_{\ell,t}$.

2. BLIND SOURCE SEPARATION USING DELAYED CORRELATION

We are often interested in a slightly more general separation where the image sources are independent regarded as time signals, but the spatial patterns associated with the sources are not necessarily orthogonal. That is, we would like to be able to perform a more general decomposition of the data matrix,

$$X = AS, \quad X_{j,t} = \sum_{\ell=1}^{s} A_{j,\ell} S_{\ell,t}, \qquad (2)$$

where A is a general mixing matrix of dimension (P, s) and S is a source data matrix with dimension (s, N) consisting of $s \le P$ independent sources, i.e., $\forall t_1, t_2 :$ $E\{S_{k,t_1} S_{\ell,t_2}\} = \rho_\ell(t_1, t_2) \cdot \delta_{k,\ell}$ where ρ_ℓ are time-correlation functions and δ is the Kronecker delta. In comparison with PCA, S corresponds to the Principal components. The columns of the mixing matrix, viz. the spatial patterns associated with the sources, corresponds to the eigenimages in PCA.

Finding A, S is often referred to as Independent Component Analysis (ICA), see e.g., [3], [5]. Rather complicated, iterative, non-linear schemes have been invoked in order to perform ICA, e.g., using higher order statistics or non-linear neural net algorithms, see e.g., [7].

Molgedey and Schuster [8] have proposed a source separation scheme which, in contrast to standard ICA [3], is based on assuming non-vanishing (time)autocorrelation functions of the independent sources. The Molgedey-Schuster scheme uses time-lagged information about the sources like alternative source separation methods [1], [2], [9], however, is advantageous as being a non-iterative "one-shot" algorithm.

2.1. Original Molgedey-Schuster Algorithm

In [8] the algorithm was developed for sources mixed by a square, non-singular, A-matrix. Here we generalize their approach to a singular case where the number of sources is much smaller than the number of pixels (channels), i.e., $s \ll P$.

Let X_τ be the time shifted data matrix, e.g., by cyclic permutation by τ time steps, that is, $X_{\tau,j,t} = X_{j,(t+\tau) \bmod N}$. The delayed correlation approach is based on solving the simultaneous eigenvalue problem for the correlation matrices $X_\tau X^\top$ and $XX^\top$. This is implemented by solving the eigenvalue problem for the *quotient* matrix

$$Q = X_\tau X^\top (XX^\top)^{-1}. \tag{3}$$

If there exists A, S so that $X = AS$, we find

$$XX^\top = ASS^\top A^\top, \quad X_\tau X^\top = AS_\tau S^\top A^\top. \tag{4}$$

Since the sources are independent and time-correlation ergodic, we obtain $\lim_{N\to\infty} N^{-1}SS^\top = C(0)$, equal to the diagonalsource crosscorrelation matrix at lag zero. Similarly, $\lim_{N\to\infty} N^{-1}S_\tau S^\top = C(\tau)$ produces the diagonal crosscorrelation matrix at lag τ. Hence, to zero order in $1/N$,

$$\begin{aligned} X_\tau X^\top \left(XX^\top\right)^{-1} &= AC(\tau)A^\top (A^\top)^{-1} C(0)^{-1} A^{-1} \\ &= AC(\tau)C(0)^{-1}A^{-1} \end{aligned} \tag{5}$$

with $C(\tau)C(0)^{-1}$ being a diagonal matrix. If we solve the eigenvalue problem for the quotient matrix Q we have a direct scheme for estimating A, S,

$$Q\Phi = \Phi\Lambda. \tag{6}$$

We identify $\Phi = A$ and $\Lambda = C(\tau)C(0)^{-1}$ up to scaling factors.

2.2. Molgedey-Schuster Algorithm for Short Sequences

By *short* image sequences, we mean sequences where $N < P$. Such sequences appear in many contexts, e.g., video processing of non-stationary scenes (the mixing matrix can only be considered constant over relative short time horizons) or in the context of neuroimaging by Positron Emission Tomography (PET) or functional Magnetic Resonance Imaging (fMRI). For short image sequences we can not directly apply the delayed correlation scheme, since the correlation matrix $C(0) = XX^\top$ will be singular whenever $N < P$.

We can however use a subspace projection scheme, based on the SVD of the data matrices. First we note that if X_τ is formed by cyclic permutation, then $XX^\top = X_\tau X_\tau^\top$, hence,

$$X = UDV^\top, \quad X_\tau = UDV_\tau^\top. \tag{7}$$

In other words, the correlation matrices become,

$$\begin{aligned} XX^\top &= UDV^\top VD^\top U^\top, \quad (8) \\ X_\tau X^\top &= UDV_\tau^\top VD^\top U^\top. \quad (9) \end{aligned}$$

The appropriate subspace (Moore-Penrose) inversion of $XX^\top$, is then

$$\begin{aligned} \left(XX^\top\right)^{-1} &= UD^{-1}(V^\top V)^{-1}D^{-1}U^\top \\ &= UD^{-2}U^\top, \end{aligned} \tag{10}$$

and we find that the quotient matrix for the eigenvalue problem becomes[1],

$$\begin{aligned} X_\tau X^\top \left(XX^\top\right)^{-1} &= UDV_\tau^\top VDU^\top UD^{-2}U^\top \\ &= UDV_\tau V^\top D^{-1}U^\top. \end{aligned} \tag{11}$$

We have an option here for *regularization* by reducing the number of sources, i.e., by reducing the dimension of the SVD representation. The short sequence version of the Molgedey-Schuster scheme can be summarized, in the following steps,

$$\begin{aligned} X &= UDV^\top & (12) \\ M &= V_\tau V^\top & (13) \\ M\Psi &= \Psi\Delta & (14) \\ A &= UD\Psi & (15) \\ A^{-1} &= \Psi^{-1}D^{-1}U^\top & (16) \\ S &= A^{-1}X = \Psi^{-1}V^\top. & (17) \end{aligned}$$

The fact that Ψ^{-1} is non-orthogonal in general implies that $\widehat{C}(0) = N^{-1}SS^\top = \Psi^{-1}(\Psi^{-1})^\top$ and $\widehat{C}(\tau) = N^{-1}S_\tau S^\top = \Delta\Psi^{-1}(\Psi^{-1})^\top$ will not be diagonal, i.e., that the finite sequence source signals S can not be expected to be *uncorrelated*. Unlike PCA, this scheme and other ICA schemes, do not automatically produce at set of uncorrelated features.

3. EXPERIMENTS

To illustrate the potential of the delayed correlation method for source separation in a specific short image sequence we create a toy example in which $s = 3$ independent periodic sources signals are mixed. The source signals are shown in Figure 2. The spatial patterns, i.e., the columns of the mixing matrix A, are created non-orthogonal, as shown the upper panel of

[1] Note that V is quadratic as $L = N$.

Figure 1. $N = 200$ images were created by mixing the three basic images. Using the delayed correlation scheme there are two parameters to set, the delay τ and the number of singular components retained in the eigenvalue problem (14). We have found that the performance of the scheme is sensitive to the choice of these parameters. A short delay was found to give the optimal decomposition, hence, $\tau = 1$ was chosen in the experiment. Further, nothing prevents the sources obtained in (17) from being complex valued, but as the matrix M is real, possible complex signals and eigenvalues will appear in complex conjugate pairs. Hence, special precaution must be taken when reconstruction signals from the individual sources found. No complex eigenvalues appeared when the number of channels was kept small (in this case below 5-7 channels). Hence, we choose to keep only three channels in the source separation problem. The "true" source sequences, the sequences reconstructed by PCA, and the sequences obtained by the delayed correlation method are shown in Figure 2. As seen in Figure 1, the PCA orthogonality constraint of the spatial pattern prevents it from separating the sources, and the resulting components, although being uncorrelated in time, do not resemble the true source time series of Figure 2.

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5. CONCLUSION

This paper presented a modified version of the Molgedey-Schuster algorithm for blind source separation in the case of short sequences of high dimensional signals. The viability of the suggested algorithm was demonstrated on a simple toy-problem of mixing 3 image sequences. The algorithm performance is sensitive to the setting of the delay parameter τ, and we are currently working on generalization based methods for automatic determination of the optimal τ. In addition, this method can be used to select the optimal number of sources.

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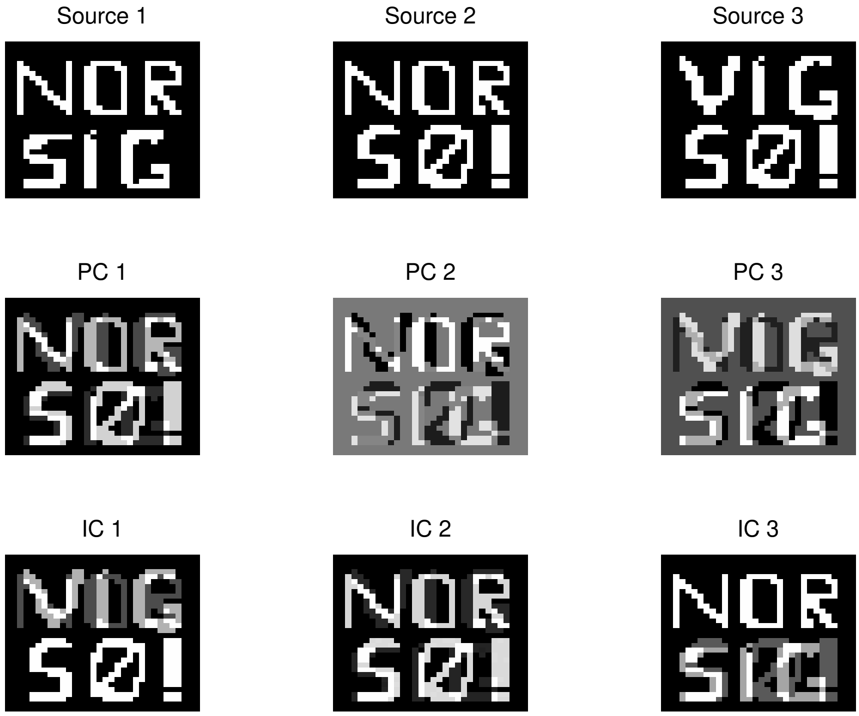

Figure 1: Upper row: Three 32×32 images corresponding to the columns of the mixing matrix A (i.e., $P = 1024$) which represent the spatial patterns associated with a specific source. Middle row: The three eigenimages found by PCA. Bottom row: The three spatial patterns of the independent components found by the delayed correlation method.

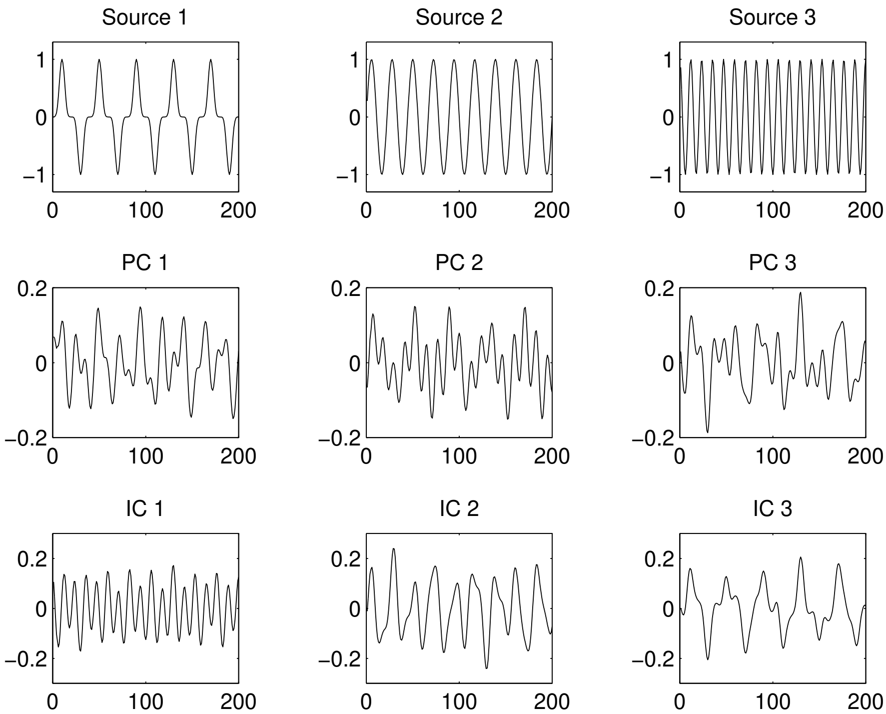

Figure 2: Upper row: The three source time series of length $N = 200$ of the toy experiment. Middle row: The Principal Components found by PCA. Bottom row: The Independent Components found by the delayed correlation method.