Restoring functional PET Images using Anatomical MR Images

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INTRODUCTION

Due to the low signal to noise ratio and the low spatial resolution in Positron Emission Tomography (PET) images, it is desired to use a-priori knowledge about the brain to address this. We present a method using a Bayesian approach to include new structural a-priori information from Magnetic Resonance (MR) images. We use a 3D Markov Random Field (MRF) model with only nearest neighbour connections and edge elements.

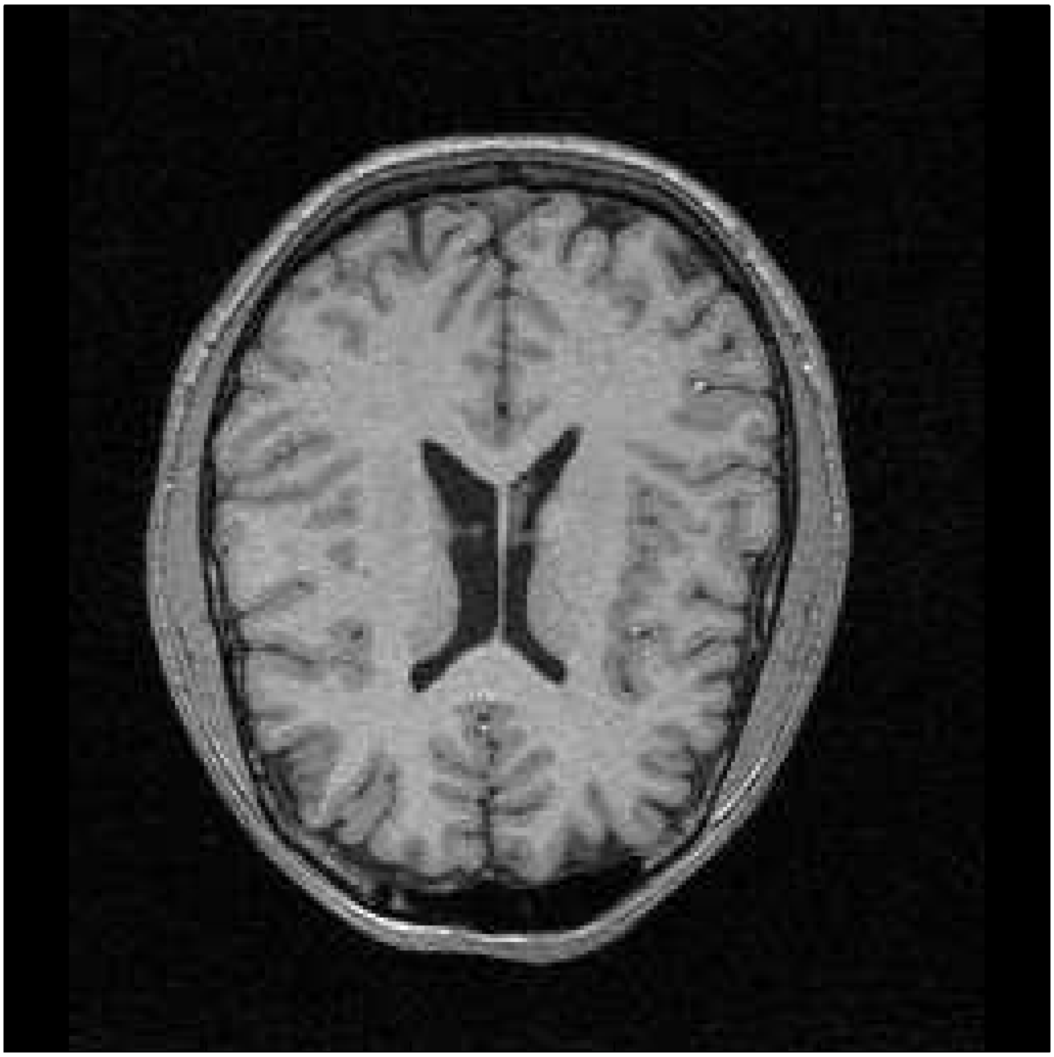
This presentation includes:

- A Few Words about Brain scans -
  What is Magnetic Resonance (MR) Imaging ?
  And Positron Emission Tomography (PET) Images?
- Segmentation of MR Images -
  Extraction of important edges.
- Markov Random Fields -
  Theory and Background.
  Sensor fusion.
- Restorations of PET images -
  Results !
- Conclusion

MAGNETIC RESONANCE (MR) IMAGING

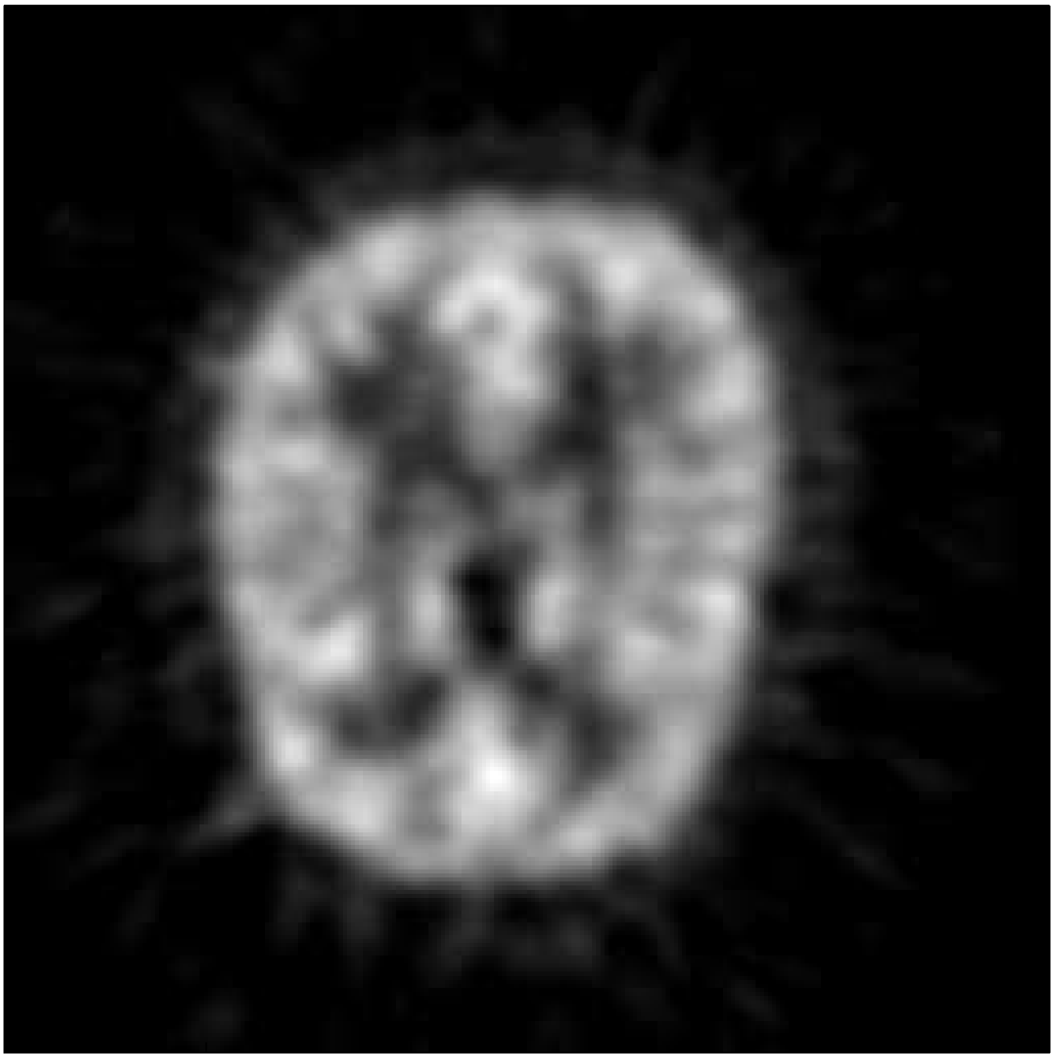
Magnetic Resonance Imaging is a technique using changes in the magnetic field to measure structural information, because the different tissue types have different magnetic responses. Using different input response function it is possible to enhance specific tissue types.

This a slice of a MR image.



POSITRON EMISSION TOMOGRAPHY (PET) IMAGING.

PET imaging is based on radioactive tracers injected in the persons blood. The tracer can e.g. be O-15 and a part of water H_2O. The PET image gives a picture of the blood flow in the brain, which is correlated with the brain activity. The slice seen below is smoothed during co-registration and sinc-interpolation, and therefore looks less noisy than normal.



SEGMENTATION OF MR IMAGES.

Segmentation of MR images is done in several steps.

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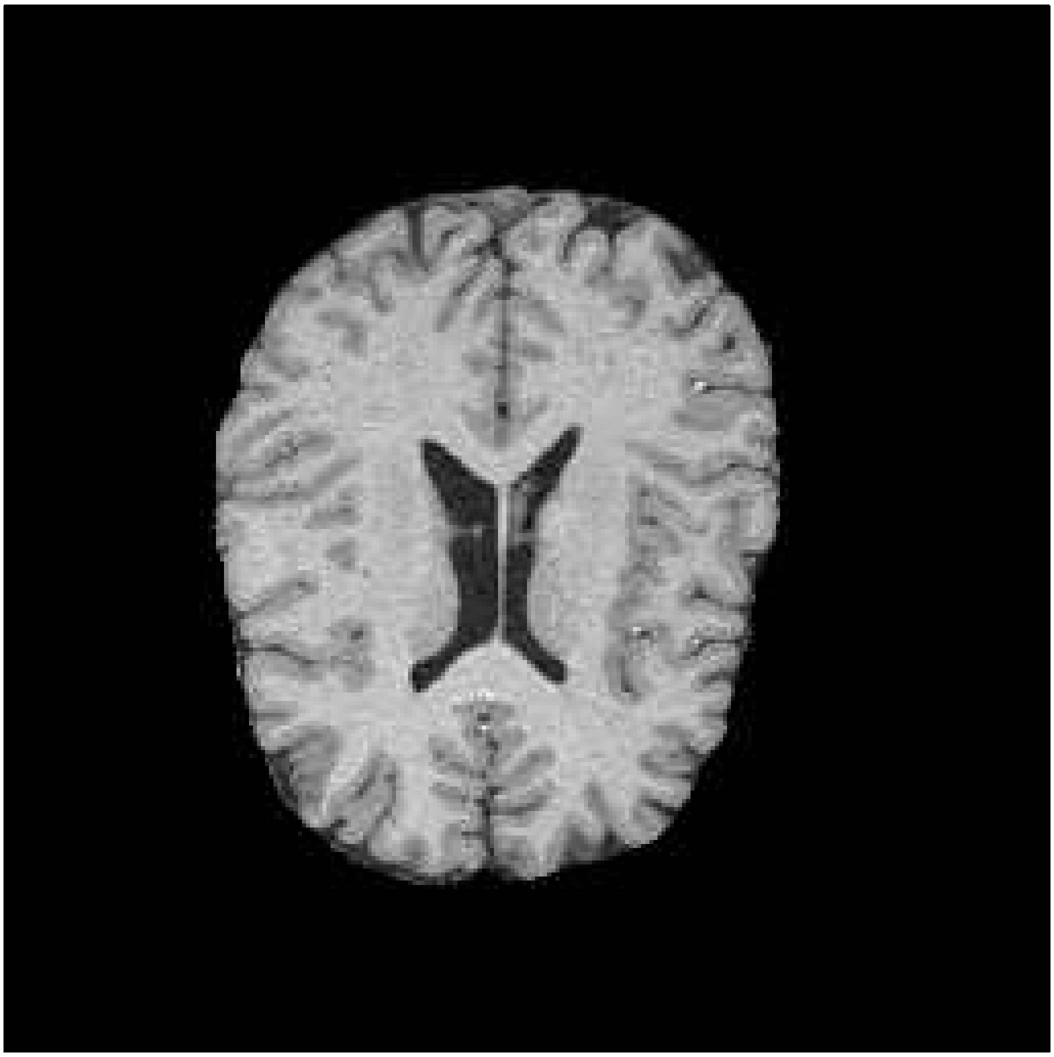
1. **Removing of Non Brain, e.g. finding the brain in the raw MR image.**
2. **Segmenting Brain into different tissue types.**
3. **Extracting edge information from segmented images.**

The first step is only done to receive a better final result and are not in theory necessary, but useful in the co-registration between PET and MR. In this example we have 2 MR images from the same person scanned with different protocols. These are assumed to be co-registered, which is a non trivial problem if the scanner don't support it.

EXTRACTING THE BRAIN FROM RAW MR IMAGES

Extraction of the Brain can be done in many different ways, depending on the needs of the subsequent analyse. In this project we used a semi automated method, developed in corporation with Brain Image Analysis Lab. (BIAL), Department of Psychiatry, University of California San Diego (UCSD).

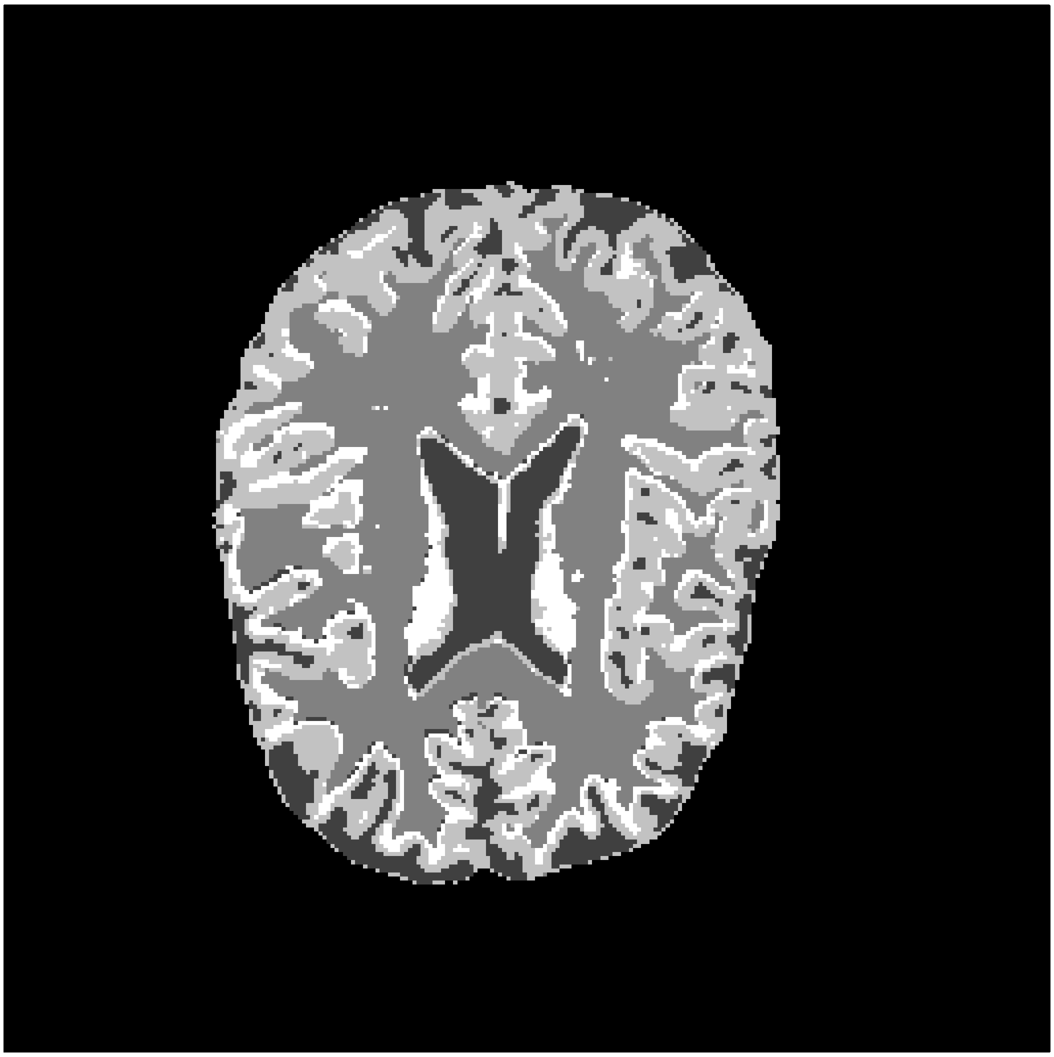
Only non-Brain tissue is removed.



SEGMENTATION OF THE BRAIN.

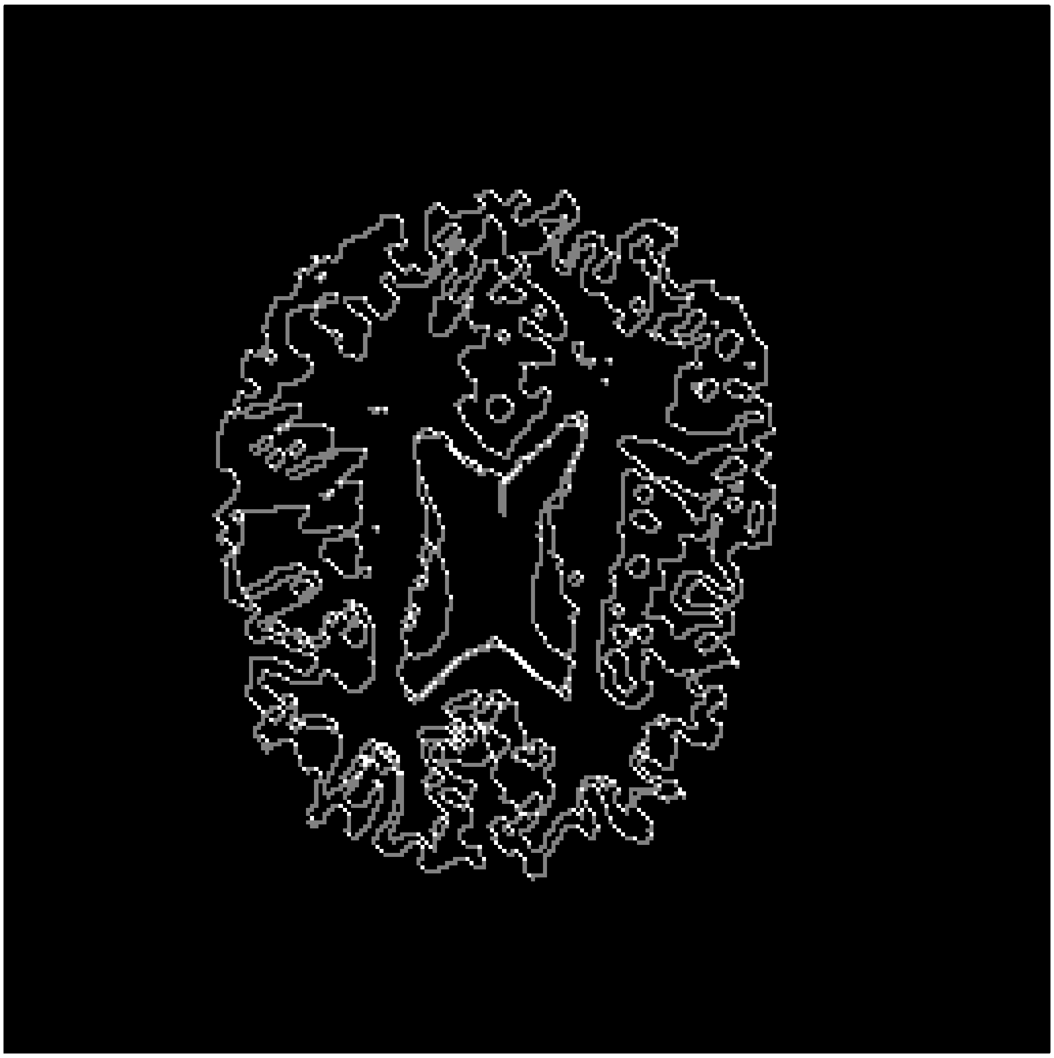
We use a histogram segmentation based on user picked samples. These samples was picked by trained professionals at BIAL, UCSD. We segment into white matter, cortex gray matter, subcortical gray matter and cerebrospinal fluid (csf).

Segmented MR image.



EXTRACTING EDGE INFORMATION FROM SEGMENTED MRI.

In PET images mainly gray matter and white matter contribute with activity and there are no difference between the 2 types of gray matter (subcortical and cortical).



SYSTEM MODEL - BAYESIAN DESCRIPTION OF VISUAL SYSTEMS

The basic idea is to consider both the observed signal and the processes of the imaging system as stochastic processes. The Bayes formula can then be used to obtain the distribution $P(\mathbf{V}|\mathbf{d})$ of the reconstructed signal $\mathbf{V}$, conditioned on the observed degraded signal $\mathbf{d}$:

$$P(\mathbf{V}|\mathbf{d}) = \frac{P(\mathbf{d}|\mathbf{V})P(\mathbf{V})}{P(\mathbf{d})}$$

This conditional distribution is the product of the distribution of the imaging system process, also called the observation model: $P(\mathbf{d}|\mathbf{V}) \equiv P(\mathbf{V} \rightarrow \mathbf{d})$, and the *prior* distribution of the reconstructed signal $P(\mathbf{V})$. $P(\mathbf{V}|\mathbf{d})$ is referred to as the *posterior* distribution. Under a given observation, $\mathbf{d}$, $P(\mathbf{d})$ is constant.

A meaningful estimate of the reconstructed signal is given by the *Maximum A Posterior* estimate.

The Prior model of the system is in this context a Markov Random Field Model containing both the intensity values and edge labels.

MARKOV RANDOM FIELD AS PRIOR.

In this section there is a description of the prior distribution a Gibbs-Markov distribution.
Introducing edges p_i, q_i and r_i connected to edges in the x, y and z direction. The binary labels for the edges is

$$\{Edge, NonEdge\} \text{ , implemented as } \{+1, -1\}.$$

The edge element p_i have the label $\{Edge\}$ if the signal elements V_i and V_{i+dx} are independent.

In 2D the structure looks like this

$V_{i-dx+dy}$	$p_{i-dx+dy}$	V_{i+dy}	p_{i+dy}	$V_{i+dx+dy}$
q_{i-dx}		q_i		q_{i+dx}
V_{i-dx}	p_{i-dx}	V_i	p_i	V_{i+dx}
$q_{i-dx-dy}$		q_{i-dy}		$q_{i+dx-dy}$
$V_{i-dx-dy}$	$p_{i-dx-dy}$	V_{i-dy}	p_{i-dy}	$V_{i+dx-dy}$

The Gibbs distribution have the following form:

$$P(\mathbf{V}, \mathbf{p}, \mathbf{q}, \mathbf{r} \mid \theta) = Z_1^{-1} \exp\left(-E_{\mathrm{prior}}(\mathbf{V}, \mathbf{p}, \mathbf{q}, \mathbf{r}|\theta)\right),$$

where $E_{\mathrm{prior}}(\mathbf{V}, \mathbf{p}, \mathbf{q}, \mathbf{r}|\theta)$ is the energy or cost function for the MRF model, θ are the model parameters and Z_1^{-1} is a normalisation constant. A simple model can look like this:

$$\begin{aligned} E_{\mathrm{prior}}(\mathbf{V}, \mathbf{p}, \mathbf{q}, \mathbf{r} \mid \theta) \;=\; & \frac{\nu}{2} \sum_i (1 - p_i)(V_i - V_{i+dx})^2 \\ +\; & \frac{\nu}{2} \sum_i (1 - q_i)(V_i - V_{i+dy})^2 \\ +\; & \frac{\nu}{2} \sum_i (1 - r_i)(V_i - V_{i+dz})^2 \\ +\; & \sum_i \mu_i^p p_i + \sum_i \mu_i^q q_i + \sum_i \mu_i^r r_i \end{aligned}$$

The first 3 terms are penalty terms in the x, y and z direction, neighbour intensity difference is punished if there is no active edge. The last terms is added to control the number of active edges, but they are constant with fixed edges from MR images and have no influence in simple method.

OBSERVATION MODEL.

The observation model is a simple linear transformation with added spatial white Gaussian noise.

Observations model for each point d_j of $\mathbf{d}$.

1. $d_j = \varphi_j(\mathbf{V}) + e_j$
2. where $\varphi_j(\mathbf{V}) = \Sigma_i \, \varphi_{ji} V_i$
3. and $e_j \in \mathcal{N}(0, \sigma_j^2)$ is independent for each d_j.

$\Rightarrow$ This gives $(d_j|\mathbf{V}) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left(-\frac{(d_j - \varphi_j(\mathbf{V}))^2}{2\sigma_j^2}\right)$

Each of the $p(d_j|\mathbf{V})$ is independent and therefore

$$p(\mathbf{d}|\mathbf{V}) = Z_2 \exp\left(-\sum_j \frac{(d_j - \varphi_j(\mathbf{V}))^2}{2\sigma_j^2}\right)$$

where $Z_2 = \Pi_j \frac{1}{\sqrt{2\pi\sigma_j^2}}$

FINDING MAP ESTIMATE.

To find the MAP solution a mean field approximation (MFA) is used to obtain a faster result, without estimation the normalisation constant. The MFA can be described as an approximation where all elements are replaced by their mean while updating element V_i.

$$
\begin{aligned}
\bar{V}_i \;=\; & \frac{1}{\kappa}\Bigg[\frac{1}{\nu}\sum_j \frac{1}{\sigma_j^2}\varphi_{ji}(d_j - \varphi_j(\mathbf{V})) + \frac{1}{\nu}(\sum_j \frac{1}{\sigma_j^2}\varphi_{ji}^2)V_i \\
+ \; & (1-p_i)V_{i+dx} + (1-p_{i-dx})V_{i-dx} + (1-q_i)V_{i+dy} \\
+ \; & (1-q_{i-dy})V_{i-dy} + (1-r_i)V_{i+dz} + (1-r_{i-dz})V_{i-dz}\Bigg]
\end{aligned}
$$

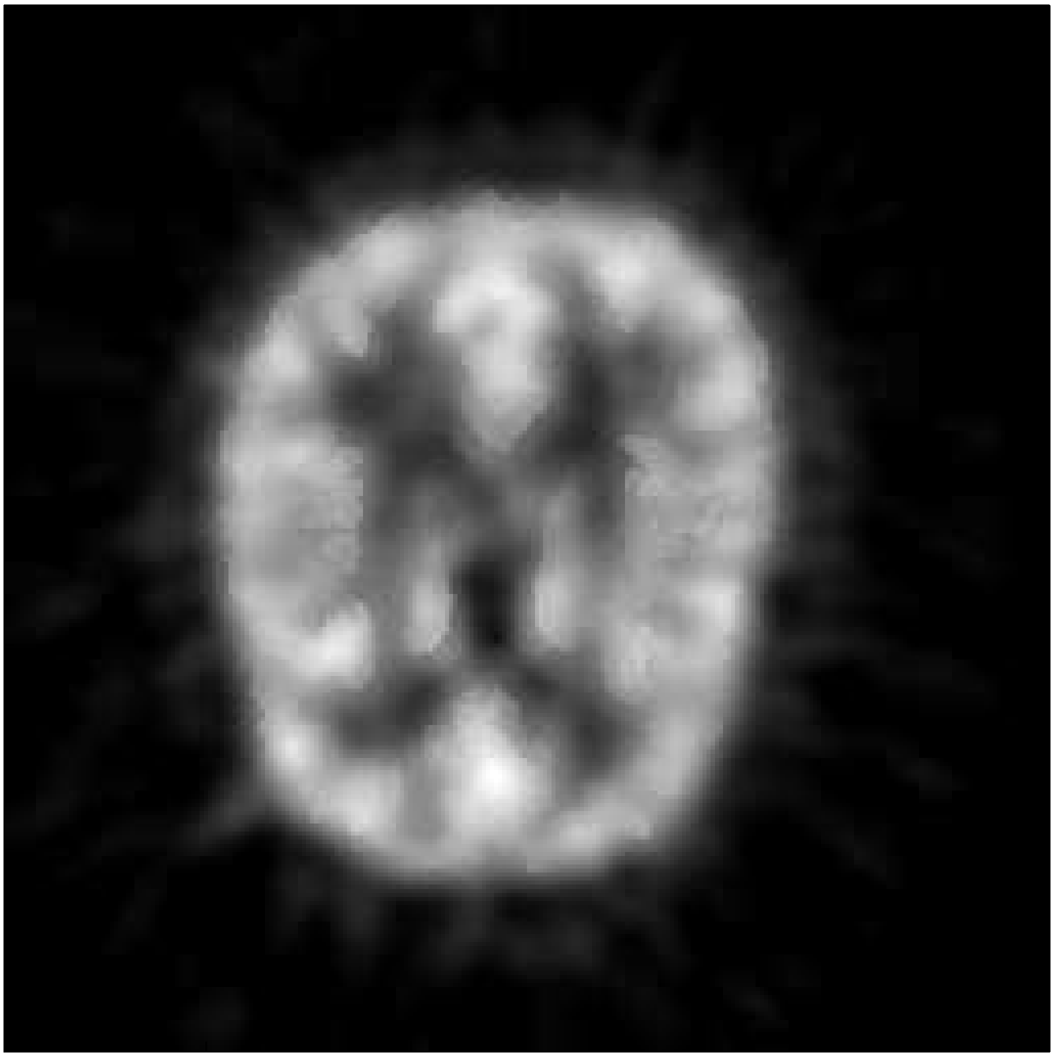
where

$$
\kappa = \frac{1}{\nu}\sum_j \frac{1}{\sigma_j^2}\varphi_{ji}^2 + 6 - p_i - p_{i-dx} - q_i - q_{i-dy} - r_i - r_{i-dz}
$$

In this problem the edges are fixed by the initialisation from the MR segmentation, but it is possible to simply extend the model to be able to create new edges which correspond to the different activation areas.

RESULTS

As seen below it is possible to restore some of the edges using information from MR images and obtain a better PET image.



CONCLUSION

+ It is possible to incorporate information from MR in PET images.
+ Obtain higher resolution and/or better signal/noise ratio.
- Result very depended on co-registration.