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Comparison of two convolution models for fMRI time series

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Abstract

- Haemodynamic Response represented by a kernel
- Two different models compared with respect to estimation of response strength and response delay

Convolution models

We represent the haemodynamic response as measured in a fMRI study with a kernel that is convolved linearly with the activation signal — the paradigm. The output from this convolution is fitted to the fMRI data with the least square criteria by optimising the kernel parameters.

$$y(u,t) = \sum_{\tau=t-T}^{t} h(u,t-\tau)x(\tau) + \eta(u,t)$$

- h Convolution kernel, representation of the haemodynamic response
- t Sample (or scan) index
- u Voxel index
- x Input, the activation (paradigm)
- y Output, the measured fMRI time series
- η Noise, assumed normal distributed
- τ Kernel lag time

Response Strength

As the response strength we use the standard deviation of the model output divided by the standard deviation on the input:

$$\text{Response Strength} = \frac{std(y(\mathbf{u}))}{std(x(\mathbf{u}))}$$

Delay

As a measure for the delay of the response we are using the center of mass of the kernel τ_m.
Other delay measures are the group delay τ_g and the 10%-90% rise time.
The center of mass is actually the group delay for the zero'th frequency component.
In the Fourier domain the group delay will have the following form:

$$\tau_g(\omega) = \frac{\frac{\partial H_r(\omega)}{\partial \omega} H_i(\omega) - \frac{\partial H_i(\omega)}{\partial \omega} H_r(\omega)}{|H(\omega)|^2}$$

- H Fourier transformed kernel
- H_r Real part of Fourier transformed kernel
- H_i Imaginary part of fourier transformed kernel
- ω Frequency

Lange-Zeger Model

In the Lange-Zeger model the output is found by convoluting the input (activation) with the *gamma density kernel*.

$$h_{\text{LZ}}(u,t) = \beta_u \theta_{2,u}^{\theta_{1,u}} (\theta_{2,u} t)^{\theta_{1,u}-1} \exp(-\theta_{2,u} t)/\Gamma(\theta_{1,u})$$

- Nonlinear in the parameters, — linear in the input
- Complex shape of kernel with small number of parameters
- Optimisation of the kernel in the frequency domain (simple derivatives) through iterative gradient descent *or* optimisation in the time domain with grid search
- Will allways implement "low-pass" filtering
- Instability due to sampling (restrictions on the parameters)
- Complicated parameter estimation due to parameter dependencies

Group delay:

$$\tau_{g,u}(\omega) = \frac{\theta_{1,u}}{\theta_{2,u}} \frac{\theta_{2,u}^2}{\theta_{2,u}^2 + \omega^2}$$

Center of mass:

$$\tau_{m,\text{LZ},u} = \frac{\theta_{1,u}}{\theta_{2,u}}$$

FIR model

The FIR model (Finite Impuls Response) is a simple regression model: The input (activation) convolved with a general kernel of finite length. The parameters (or *coefficients*) are found from the *normal equations*. The complexity of the model can be controled with a *weight decay* term (*Ridge regression*):

$$\mathbf{H} = (\mathbf{X}^{\mathsf{T}}\mathbf{X} + \lambda \mathbf{I})\mathbf{X}^{\mathsf{T}}\mathbf{Y}$$

- Linear in the parameters, — linear in the input
- Very stable estimation of the parameters
- Will implement general linear filters
- The model is able to model the onset overshoot found in some fMRI signals
- Limited lag space
- The representation of a complex kernel requires many parameters
- Problems with overfitting

Center of mass:

$$\tau_{m,\text{FIR},u} = \frac{\sum t\, h(u,t)}{\sum h(u,t)}$$

An artifact with this definition is that with some combinations of FIR parameters the center of mass will become negative.

Data

Subjects : 3 subjects.

Task : Left-handed finger-to-thumb opposition.

Modality : Whole-brain echo planar fMRI images.

Paradigm : Multiple runs of 72 images. 24 baseline, 24 activation, 24 baseline

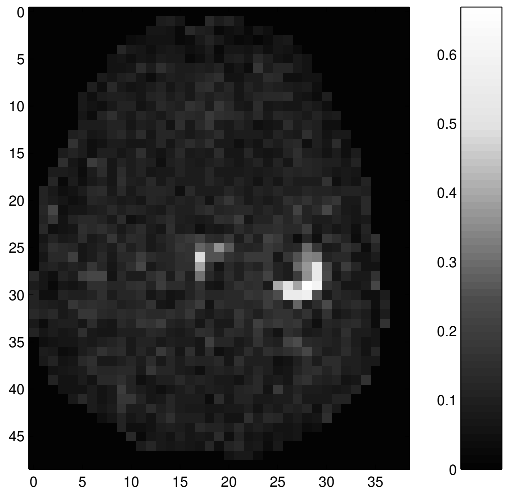
Time step : 2.5 second.

Location : Axial slice through primary motor cortex and SMA.

Area : 64x64 voxels, each 3.1x3.1x8mm.

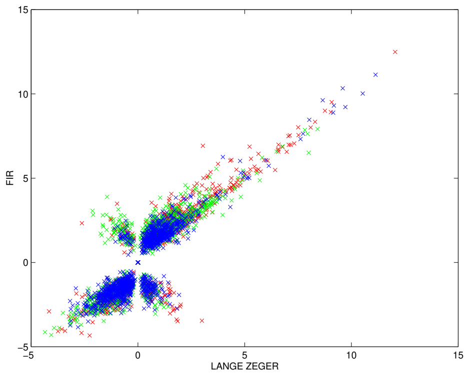
Preprocessing : Masked, motion alignment, centering

Analysis : FIR: 7 parameters ridge regression, Lange-Zeger: grid search



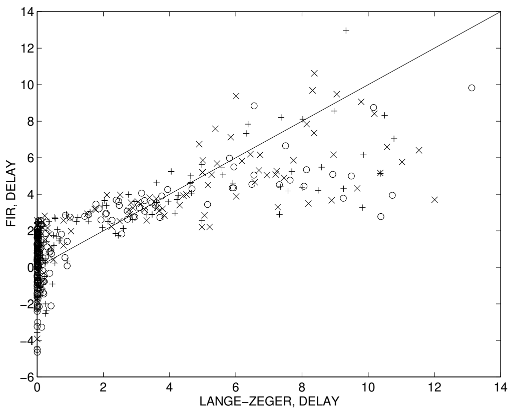
Slice through primary motor and supplementary motor area for one of the subjects showing the response strength from the FIR model. Both models capture these areas.

Response Strength



Scatter plot for the response *strength* comparison between the Lange-Zeger and the FIR model.

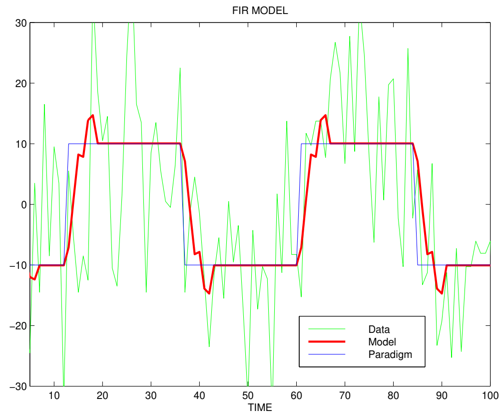
Delay



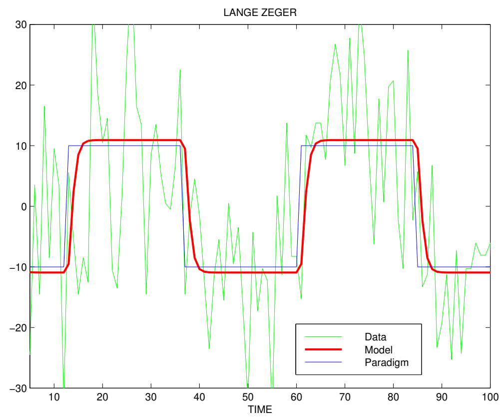
Scatter plot for the response *delay* comparison between the Lange-Zeger and the FIR model.

The Lange-Zeger model is able to model both long and short delays. The FIR model is not able to model long delays due to the limited lag space.

Kernel Fitting



Plot of a part of the FIR model response shows that the model is able to capture the overshoot of the activation.



The Lange-Zeger model is not able to track rapid changes in the fMRI time signal.

The FIR model is more flexible than the Lange-Zeger model and can model intrinsic features that (perhaps) are present in the fMRI signal.

Discussion

- A comparison between the response strength exposes *The Frying Pan* — looking at the scatter plot: Two gaussian cluster representing noise voxels and activated voxels. The connection between the two (the handle and the frying pan) is due to point spread and the finite spatial size of the voxel.
- We note that both models are linear in the input, thus the kernels will be symmetric at the onset and offset of the activation. A nonlinear model will be able to model the onset and offset independently.

Further/Ongoing Work

- Investigation of the FIR model (number of parameters, regularisation) through cross-validation
- Nonlinear regression (nonlinear in the input)

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Reference

Nick Lange and Scott Zeger: *Non-linear Fourier Time Series Analysis for Human Brain Mapping by Functional Magnetic Resonance Imaging*, 1997, 1, *Journal of Royal Statistical Society, Series C. Appl. Statist.*