Comparison of two convolution models for fMRI time series

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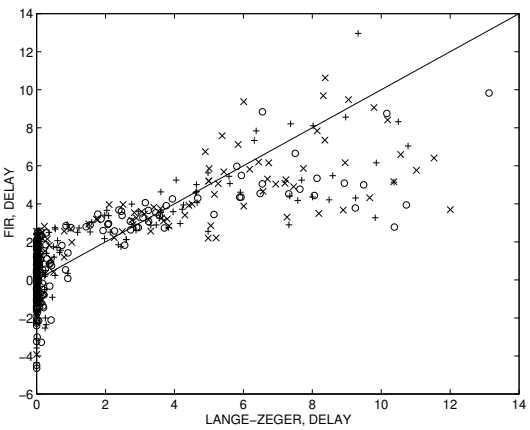
Introduction. The fMRI signal represents the hemodynamic response to focal neuronal activation. Simple convolution models of the hemodynamic response of the form $y(\mathbf{u},t) = \sum_{t'=-\infty}^{t} h(\mathbf{u},t-t')x(t') + n(\mathbf{u},t)$ are being investigated (see e.g. (1)). $y(\mathbf{u},t)$ is the local ($\mathbf{u}$) fMRI signal, while $x(t)$ is the activation reference function, $h(\mathbf{u},t-t')$ is the response convolution kernel, and $n(\mathbf{u},t)$ is random white noise. In the Lange-Zeger model (1) the response kernel is parameterized by a strength ($\beta(\mathbf{u})$) and the two parameters of a univariate Gamma distribution. By representing the response with three parameters pr. voxel, the Lange-Zeger model basically constrains the space of possible kernels to "low-pass" filters (the hemodynamic response in a given voxel is either strictly positive ($\beta(\mathbf{u}) > 0$) or strictly negative ($\beta(\mathbf{u}) < 0$)). By introducing such bias parameter variance is reduced. Here we compare the Lange-Zeger model with a model in which the components of $h(\mathbf{u},t-t')$ are considered independent free parameters to be fitted. This model too is biased, with bias controlled by the number of non-zero kernel components. The latter model represents a general Finite Impulse Response (FIR) filter.

Methods. fMRI activation image sets from three subjects performing a left-handed finger-to-thumb opposition task were acquired. Multiple runs of 72 2.5-second (24 baseline, 24 activation, 24 baseline) whole-brain echo planar images where aligned, and an axial slice through primary motor cortex and SMA of 42 x 42 voxels (3.1 x 3.1 x 8 mm) extracted. For each voxel the parameters of the Lange-Zeger model and of the FIR model were fitted, using an ordinary least squares procedure. For $x(t)$ a spatially uniform square wave activation paradigm was assumed. For the FIR model a maximum lag ($L_{\max} = 6$) was chosen (i.e. $h(\mathbf{u},\tau) \equiv 0$ for $\tau > 6$). For both models a *response strength* was extracted: $\beta(\mathbf{u})$ for the Lange-Zeger model and output/input ratio between signal standard deviations $\mathrm{std}(y(\mathbf{u}))/\mathrm{std}(x(\mathbf{u}))$ for the FIR model. Similarly a *response delay* was defined for both models as the "center of mass" of the response, $\delta(\mathbf{u}) = \sum_\tau \tau h(\mathbf{u},\tau) / \sum_\tau h(\mathbf{u},\tau)$.

Results and Conclusion. Qualitatively, we find that both models are able to pick up the expected response in primary motor area and in SMA. However, the temporal structure of the responses is somewhat different. The FIR model does exploit its flexibility to model both negative and positive parts of the kernel for most voxels. The Lange-Zeger model implements a kernel with either all positive or all negative components. Quantitatively, we find a correlation coefficient of $R \approx 0.85$ between the response strenghts of the two models. We show in the figure a scatterplot of the observed delays for the FIR model vs. the Lange-Zeger model. Delays derived for the three subjects are denoted by different symbols. Note that small and intermediate delays correlate well between the models the two models disagree more about the longer delays. The FIR model systematically estimates lower delays than the Lange-Zeger model, since the FIR kernel is of limited duration ($\mathrm{L_{max}}$). Note that the estimated delay in the FIR model can be negative for certain combinations of kernel components; this is an artifact reflecting the unbiased estimate of the non-zero components. In conclusion we note that both models are able to model salient aspects of the hemodynamic response. The FIR model provides a more flexible kernel structure at the price of a more noisy estimate, while the more robust Lange-Zeger model can fit large delays with relatively few parameters. By applying both in combination we can learn new features of the temporal structure of the fMRI signal.



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Reference

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