Mean Field Reconstruction with Snaky Edge Hints

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1 Introduction

Reconstruction of imagery of a non-ideal imaging system is a fundamental aim of computer vision. Geman and Geman [Geman(1984)] introduced Metropolis sampling from Gibbs distributions as a simulation tool for visual reconstruction and showed that a *Simulated Annealing* strategy could improve the efficiency of the sampling process. The sampling process is implemented as a stochastic neural network with symmetric connections. Peterson and Anderson [Peterson(1987)] applied the *Mean Field* approximation, and observed substantial improvements in speed and performance.

In the next section the Bayesian approach to reconstruction is outlined and we study the so-called *Weak Membrane* model, as a model example. The Weak Membrane is a popular vehicle for piece wise smooth reconstruction and involves edge units (called line processes in [Geman(1984)]). Edge unit control has shown to be a major challenge in applications of the Weak Membrane. However, recently there has been substantial progress in use of deformable models for contour (2D) and surface (3D) modeling, see, e.g., [Cohen(1993), Chen(1994)]. In this contribution we suggest to combine the two approaches, in particular we show how a "Snake" contour model may be used to produce efficient edge hints to the Weak Membrane. Finally, section three contains experiments and concluding remarks.

2 Bayesian Visual Reconstruction

The basic idea is to consider both the source (un-degraded) signal and the processes of the imaging system as stochastic processes. The Bayes formula can then be used to obtain the distribution $P(V|d)$ of the reconstructed signal

V, conditioned on the observed degraded signal d:

$$P(V|d) = \frac{P(d|V)P(V)}{P(d)} \tag{1}$$

This conditional distribution is the product of the distribution of the imaging system process: $P(d|V) \equiv P(V \to d)$, and the *prior* distribution of the reconstructed signal $P(V)$. $P(V|d)$ of Equation (1) is referred to as the *posterior* distribution. A useful estimate of the reconstructed signal is given by the location of the mode of the posterior distribution, the so-called *Maximum A Posteriori* estimate.

2.1 The Weak Membrane Model

The *prior* distribution reflects our general insight into the objects being imaged, for example expressing that the image represents extended 3D structures etc. The Weak Membrane is a simple model for reconstruction of piece-wise continuous intensity surfaces (2D signals) from noisy observations [Geman(1984)]. In the lattice $(m,n) \in Z^2$ version the reconstructed surface is described by the intensity values $V_{m,n}$. The prior information formalizes the expectation that neighbor intensity values should be close, except when they are disconnected by the rare occasion of an active edge-unit. We introduce horizontal edges $h_{m,n}$ and vertical edges $v_{m,n}$. The *prior* probability distribution for a membrane reads:

$$P(V,h,v) = Z_1^{-1} \exp\left(-E_{\text{prior}}(y,h,v)\right) \tag{2}$$

where $E_{\text{prior}}(y,h,v)$ is the energy or cost function:

$$\begin{aligned} & E_{\text{prior}}(V,h,v) = \\ & \frac{\nu}{2}\left[\sum_{m,n}(1-h_{m,n})(V_{m,n}-V_{m+1,n})^2 + (1-v_{m,n})(V_{m,n}-V_{m,n+1})^2\right] \\ + \quad & \sum_{m,n}\mu^h_{m,n}h_{m,n} + \sum_{m,n}\mu^v_{m,n}v_{m,n} - \sum_{m,n}\gamma_{m,n}h_{m,n}h_{m,n+1}v_{m,n}v_{m+1,n} \end{aligned} \tag{3}$$

and Z_1^{-1} is a normalization constant. This is a *Gibbs* distribution[1]. Note that the last terms in the cost function act as local *chemical potentials* for control of the number of active edge-units (i.e., $h_{m,n} = +1$).

[1]A distribution of the form $P(x) = Z^{-1}\exp(-E(x)/T)$, where $E(x)$ is a cost-function, bounded from below, and T is a parameter.

The degradation process produces the measurements ($V_{m,n} \to d_{m,n}$). In the Weak Membrane example we will for simplicity study addition of zero-mean, white Gaussian noise [2] with variance σ^2 :

$$P[d|V] = Z_2^{-1} \exp\left(-\frac{1}{2\sigma^2} \sum_{m,n} (V_{m,n} - d_{m,n})^2 \right) \tag{4}$$

Using Bayes formula (1), we can combine Equations (2) and (4), to obtain the parameterized posterior distribution:

$$P[V,h,v|d] = Z^{-1} \exp(-E(V,h,v,d)) \tag{5}$$

where Z is a normalization constant and the energy function is given by

$$E(V,h,v,d) = \frac{1}{2\sigma^2} \sum_{m,n} (V_{m,n} - d_{m,n})^2 + E_{\text{prior}}(V,h,v). \tag{6}$$

2.2 Network design

Inspecting the posterior distribution we note that the three-component process (V,h,v) realizes a *Compound Random Markov Field.* This property ensures that the neural network implementation only involves neighbor connectivity. To enhance the sampling efficiency we use a simulated annealing strategy as recommended by Geman and Geman [Geman(1984)]. The scheme is implemented by introducing a *temperature* T in the Gibbs distribution and designing the sampler with a decreasing sequence of temperatures endinged at $T = 1$. The temperature dependent distribution reads:

$$P[V,h,v|d] = Z_T^{-1} \exp\left(-\frac{E[V,h,v,d]}{T} \right) \tag{7}$$

As investigated by Geman and Geman, sampling of this distribution is too slow for most applications. It is therefore recommended to invoke a deterministic approximation scheme to obtain the necessary averages [Blake(1989), Peterson(1987)]. The self-consistent Mean Field equations for the Weak Membrane model in the k'th iteration read (see also [Hertz(1991)] for an introduction):

$$\begin{aligned}
V_{m,n}^{k+1} &= \kappa^{-1} \left[\sigma^{-2} d_{m,n} + (1 - h_{m,n}^k) V_{m+1,n}^k + (1 - h_{m-1,n}^k) V_{m-1,n}^k \right. \\
&\quad \left. + (1 - v_{m,n}^k) V_{m,n+1}^k + (1 - v_{m,n-1}^k) V_{m,n-1}^k \right] \\
h_{m,n}^{k+1} &= \tanh \left[\beta \left(\frac{1}{2} (V_{m,n}^k - V_{m+1,n}^k)^2 - \mu_{m,n}^h - \gamma_{m,n} \Gamma_{m,n}^h(k) \right) \right] \\
v_{m,n}^{k+1} &= \tanh \left[\beta \left(\frac{1}{2} (V_{m,n}^k - V_{m,n+1}^k)^2 - \mu_{m,n}^v - \gamma_{m,n} \Gamma_{m,n}^v(k) \right) \right]
\end{aligned} \tag{8}$$

[2] It is possible to incorporate any other model of the degeneration process.

where $\beta = 1/T$ and

$$\Gamma^h_{m,n}(k) = h^k_{m,n+1} v^k_{m,n} v^k_{m+1,n} + h^k_{m,n-1} v^k_{m,n-1} v^k_{m+1,n-1} \tag{9}$$

$$\Gamma^v_{m,n}(k) = v^k_{m+1,n} h^k_{m,n} h^k_{m,n+1} + v^k_{m-1,n} h^k_{m-1,n} h^k_{m-1,n+1} \tag{10}$$

$$\kappa = \sigma^{-2} + 4 - h^k_{m,n} - h^k_{m-1,n} - v^k_{m,n} - v^k_{m,n-1} \tag{11}$$

We have used the same symbols for the averaged quantities as for the stochastic ones, but that should not lead to confusion since the two never occur in the same expression. The coupled equations are solved by straightforward iteration as shown in Equation (8), defining a recursive nearest neighbor connected *cellular neural network*.

2.3 Snake hints

There has been much recent progress in the use of deformable models for edge and surface identification, see, e.g., [Cohen(1993)]. In this presentation we suggest to produce strong edge hints for the Weak Membrane through a Snake deformable model. The Snake is defined to be a periodic set of N points in the visual field of an image: $\mathbf{r}_j = (x_j, y_j)$. Periodicity meaning $\mathbf{r}_{j+pN} = \mathbf{r}_j, p \in Z$. The Snake energy function consists of a form control part E_{form} and a match part E_{match}.

$$E = E_{\text{form}} + E_{\text{match}} \tag{12}$$

In this simple version the form control preserves total length,

$$E_{\text{form}} = \frac{a}{2}(d - d_0)^2 \tag{13}$$

$$d = \sum_j |\mathbf{r}_j - \mathbf{r}_{j+1}| \tag{14}$$

where d_0 is the initial Snake length and a is a parameter to control the balance between the two parts in the energy function. More complex form controls can be designed that preserve shape, corners, etc. The Snake match energy is designed to ensure that the Snake points track edge contours in the image field. This effect may be obtained by letting the Snake points seek local maxima in the gradient energy map of the image, $G(\mathbf{r})$, evaluated from the observed image. The total Snake match energy is then given by,

$$E_{\text{match}} = -\sum_j G(\mathbf{r}_j). \tag{15}$$

Snake dynamics is established through gradient descent,

$$\frac{1}{\eta}\frac{\partial \mathbf{r}_j}{\partial t} = -\frac{\partial E}{\partial \mathbf{r}_j} = a(d - d_0)\frac{\partial d}{\partial \mathbf{r}_j} + \frac{\partial G(\mathbf{r}_j)}{\partial \mathbf{r}_j} \tag{16}$$

where η is a parameter to control the step size in the gradient direction.

3 Experiments and concluding remarks

In numerous image processing applications quite detailed prior information can be devised. In, e.g., brain scan reconstruction detailed atlases are known describing the generic brain topography under various scanning modes. To illustrate the potential of using strong edge hints in brain scans, we want to reconstruct a head phantom shown in Figure 1. This is done from a noise corrupted image shown in Figure 2. During the reconstruction process we also want to estimate edges, due to the interpretation of the image.

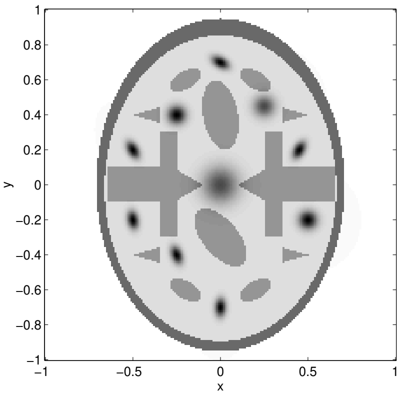

Figure 1: Original head phantom without noise.

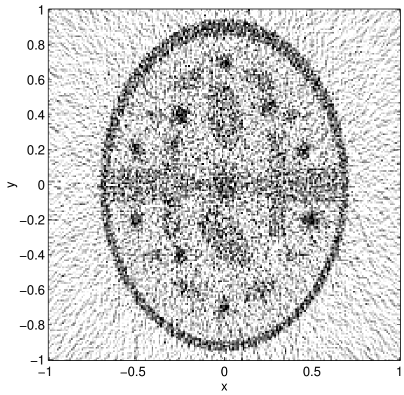

Figure 2: Noise corrupted head phantom shown in same color scale.

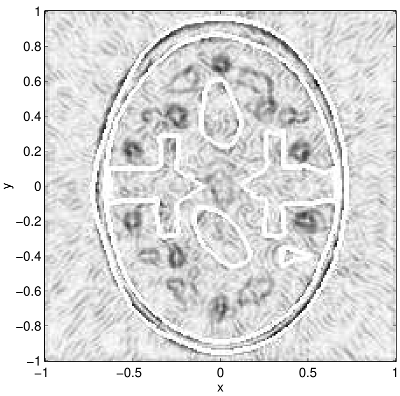

Figure 3: Edge energy map with initial snake positions shown in white.

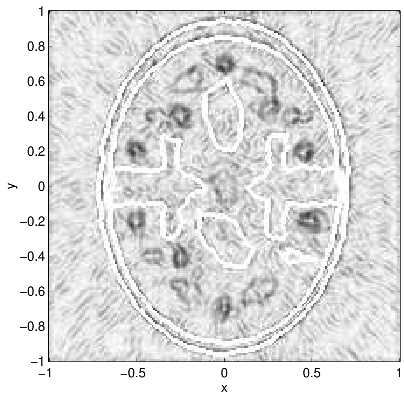

Figure 4: Edge energy map with equilibrated Snake positions shown in white.

Snakes are initialized in seven generic positions, as shown in Figure 3. For this particular instance the Snake equilibrates in about 100 iterations as illustrated in Figure 4. The approach to equilibrium is quite sensitive to the topography of the image and careful control of η is necessary, Equation (16). This problem may be relieved by use of a pseudo-second order search direction instead of the gradient of the edge energy map.

Subsequently, hints are created by modulating the chemical potentials $\mu^h_{m,n}$, $\mu^v_{m,n}$ and $\gamma_{m,n}$. This modulation implies that edge units are strongly suppressed outside of the region suggested by the Snakes. Finally, we relax the cellular network as defined in Equation (8). The result of the cellular net is presented in Figures 5 (without Snakes) and 6 (using Snakes).

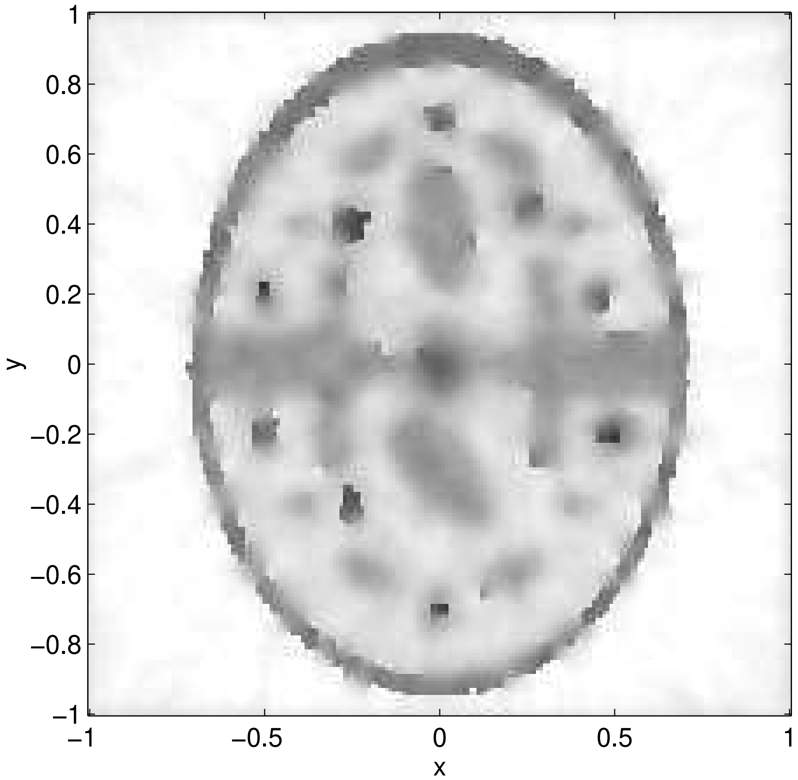

Figure 5: The restored head phantom without using Snakes edge priors to the Mean Field Annealing.

Note that the intensity levels inside the regions of the Snakes have been estimated more closely using Snakes. This might be of significant importance in, e.g., brain activation studies, in which the minute differences in brain activity between activated and resting states are investigated. Using Snakes and Mean Field Annealing enhances the Signal to Noise ratio (SNR) with 12.5 dB. Without Snakes the improvement is 11.3 dB. The improvement is due to a better estimation of edges.

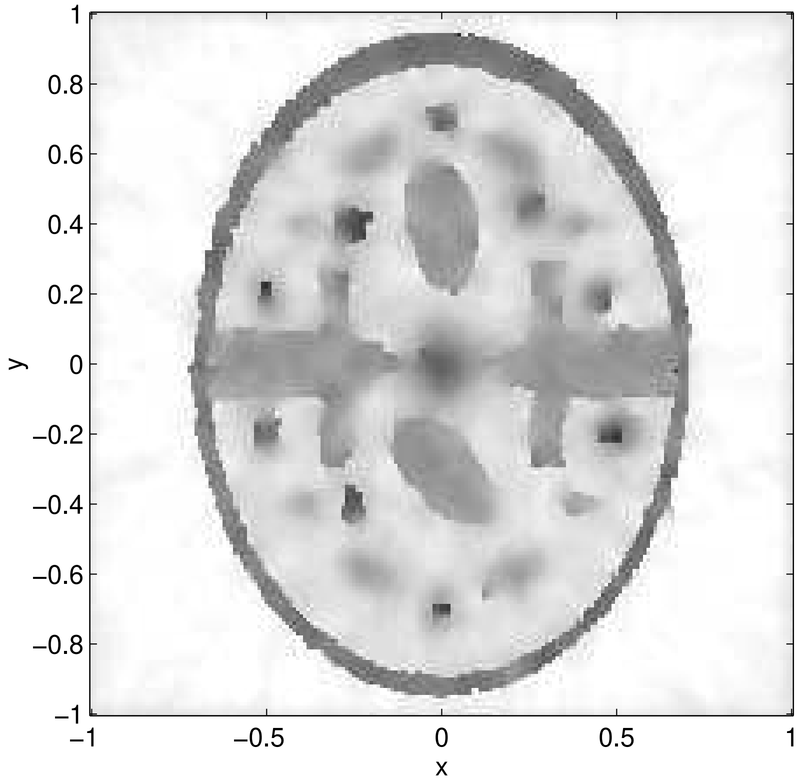

Figure 6: The restored head phantom using Snakes as edge priors to the Mean Field Annealing.

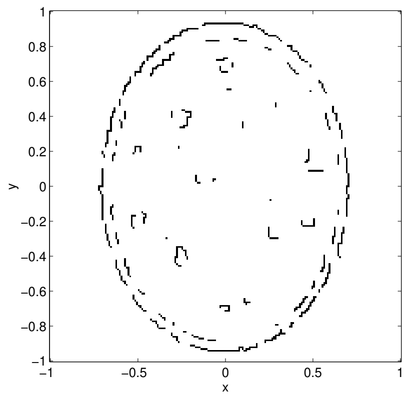

Figure 7: The estimated edges without using Snakes. Active edges are marked black.

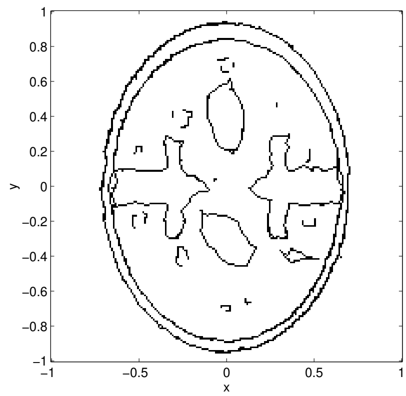

Figure 8: Estimated edges using Snakes as priors. Active edges are marked black.

In Figures 7 and 8 are shown the active edges (i.e., $h_{m,n} = +1$ or $v_{m,n} = +1$) without using Snakes respectively with Snakes. As seen from Figure 8 it is possible to incorporate strong priors in order to obtain closed edge contours. This can be used to segment the image.

In conclusion we have shown that strong structural priors may be introduced in the Weak Membrane model by invoking deformable models like Snakes. The present study was based on a head phantom; we are currently pursuing the viability of the approach in the context of Positron Emission Tomography.

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